Concept # 1: Overlapping Sets – 2 variables (2 way matrix approach)

1.

For an overlapping set problem we can use a double-set matrix to organize our information and solve. Let's call P the number of people at the convention. The **boldface** entries in the matrix below were given in the question. For example, we are told that one sixth of the attendees are female students, so we put a value of $P/6$ in the female students cell.

	FEMALE	NOT FEMALE	TOTALS
STUDENTS	**$P/6$**	$P/6$	**$P/3$**
NON STUDENTS	$P/2$	**150**	$2P/3$
TOTALS	**$2P/3$**	$P/3$	**P**

The non-boldfaced entries can be derived using simple equations that involve the numbers in one of the "total" cells. Let's look at the "Female" column as an example. Since we know the number of female students ($P/6$) and we know the total number of females ($2P/3$), we can set up an equation to find the value of female non-students:

$P/6$ + *Female Non Students* = $2P/3$.
Solving this equation yields: *Female Non Students* = $2P/3 - P/6 = P/2$.

By solving the equation derived from the "NOT FEMALE" column, we can determine a value for P.
$P/6 + 150 = P/3$ so $P = 900$
The correct answer is E.

2.

This question involves overlapping sets so we can employ a double-set matrix to help us. The two sets are speckled/rainbow and male/female. We can fill in 645 for the total number of total speckled trout based on the first sentence. Also, we can assign a variable, x, for female speckled trout and the expression $2x + 45$ for male speckled trout, also based on the first sentence.

	Male	Female	Total
Speckled	$2x + 45$	x	645
Rainbow			
Total			

We can solve for x with the following equation: $3x + 45 = 645$. Therefore, $x = 200$.

	Male	Female	Total
Speckled	445	200	645
Rainbow			
Total			

If the ratio of female speckled trout to male rainbow trout is 4:3, then there must be 150 male rainbow trout. We can easily solve for this with the below proportion where *y* represents male rainbow trout:

$$\frac{4}{3} = \frac{200}{y}$$

Therefore, $y = 150$. Also, if the ratio of male rainbow trout to all trout is 3:20, then there must be 1000 total trout using the below proportion, where *z* represents all trout:

$$\frac{3}{20} = \frac{150}{z}$$

	Male	Female	Total
Speckled	445	200	645
Rainbow	150		
Total			1000

Now we can just fill in the empty boxes to get the number of female rainbow trout.

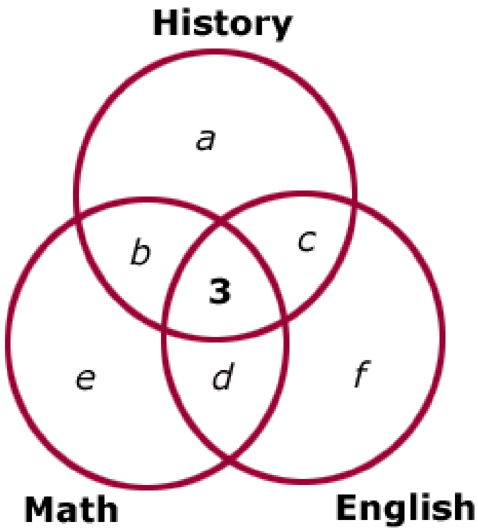
	Male	Female	Total
Speckled	445	200	645
Rainbow	150	205	355
Total			1000

The correct answer is D.

Concept # 2: Overlapping Sets – 3 variables (3 circles approach)

3.

For an overlapping set problem with three subsets, we can use a Venn diagram to solve.



Each circle represents the number of students enrolled in the History, English and Math classes, respectively. Notice that each circle is subdivided into different groups of students. Groups a, e, and f are comprised of students taking only 1 class. Groups b, c, and d are comprised of students taking 2 classes. In addition, the diagram shows us that 3 students are taking all 3 classes. We can use the diagram and the information in the question to write several equations:

History students: $a + b + c + 3 = 25$
Math students: $e + b + d + 3 = 25$
English students: $f + c + d + 3 = 34$
TOTAL students: $a + e + f + b + c + d + 3 = 68$

The question asks for the total number of students taking exactly 2 classes. This can be represented as $b + c + d$.

If we sum the first 3 equations (History, Math and English) we get:

$a + e + f + 2b + 2c + 2d + 9 = 84$.

Taking this equation and subtracting the 4th equation (Total students) yields the following:

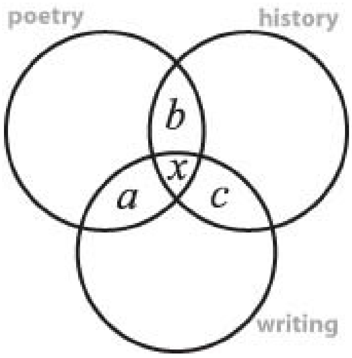
$$\begin{array}{r} a + e + f + 2b + 2c + 2d + 9 = 84 \\ \underline{-[a + e + f + b + c + d + 3 \;\; = 68]} \\ b + c + d = 10 \end{array}$$

The correct answer is B.

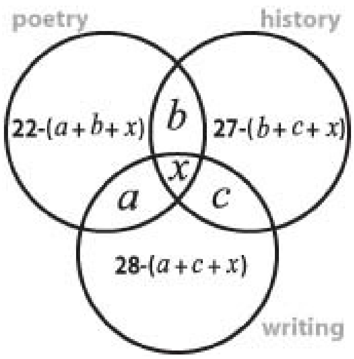
4. This is a three-set overlapping sets problem. When given three sets, a Venn diagram can be used. The first step in constructing a Venn diagram is to identify the three sets given. In this case, we have students signing up for the poetry club, the history club, and the

writing club. The shell of the Venn diagram will look like this:

When filling in the regions of a Venn diagram, it is important to work from inside out. If we let *x* represent the number of students who sign up for all three clubs, *a* represent the number of students who sign up for poetry and writing, *b* represent the number of students who sign up for poetry and history, and *c* represent the number of students who sign up for history and writing, the Venn diagram will look like this:



We are told that the total number of poetry club members is 22, the total number of history club members is 27, and the total number of writing club members is 28. We can use this information to fill in the rest of the diagram:



We can now derive an expression for the total number of students by adding up all the individual segments of the diagram. The first bracketed item represents the students taking two or three courses. The second bracketed item represents the number of students in only the poetry club, since it's derived by adding in the total number of poetry students and subtracting out the poetry students in multiple clubs. The third and fourth bracketed items represent the students in only the history or writing clubs respectively.

$59 = [a + b + c + x] + [22 - (a + b + x)] + [27 - (b + c + x)] + [28 - (a + c + x)]$
$59 = a + b + c + x + 22 - a - b - x + 27 - b - c - x + 28 - a - c - x$
$59 = 77 - 2x - a - b - c$
$59 = 77 - 2x - (a + b + c)$

By examining the diagram, we can see that $(a + b + c)$ represents the total number of students who sign up for two clubs. We are told that 6 students sign up for exactly two clubs. Consequently:

$59 = 77 - 2x - 6$
$2x = 12$
$x = 6$

So, the number of students who sign up for all three clubs is 6.

Alternatively, we can use a more intuitive approach to solve this problem. If we add up the total number of club sign-ups, or registrations, we get 22 + 27 + 28 = 77. We must remember that this number includes overlapping registrations (some students sign up for two clubs, others for three). So, there are 77 registrations and 59 total students. Therefore, there must be 77 – 59 = 18 duplicate registrations.

We know that 6 of these duplicates come from those 6 students who sign up for exactly two clubs. Each of these 6, then, adds one extra registration, for a total of 6 duplicates. We are then left with 18 – 6 = 12 duplicate registrations. These 12 duplicates must come from those students who sign up for all three clubs.

For each student who signs up for three clubs, there are two extra sign-ups. Therefore, there must be 6 students who sign up for three clubs:

12 duplicates / (2 duplicates/student) = 6 students

Between the 6 students who sign up for two clubs and the 6 students who sign up for all three, we have accounted for all 18 duplicate registrations.

The correct answer is C.

Concept # 3: Percentages – (MIND THE LANGUAGE) – these questions test more of reading ability and less of math – read very carefully in such questions:

1.
2%A+(100+1%A)/2=50+A/40

2.
Let the least one is x. When other 10 populations have the greatest value, x will have the minimum value.
X+10*1.1X=132000
X=11000
Answer is D

3.
60%*120%/(40%+60%*120%)=64%

4.
Fat in milk is x*1%, y*2% and z*3%, respectively.
So we have the equation: x*1%+y*2%+z*3%=(x+y+z)*1.5%
Simplify the equation, we can obtain that x=y+3z

5.
For 1, the tip for a $15 bill will be $2, which is less than $15*15%=2.25; the tip for a $20 will be $4, which is greater than $15*15%=2.25. Insufficient.
For 2, tips is $8, means the tens digit of the bill is 4, and the largest possible value of the bill is $49. $8>49*15%=7.35. Sufficient alone.
Answer is B

6.
Let's denote the formula for the money spent on computers as $pq = b$, where
p = price of computers
q = quantity of computers
b = budget

We can solve a percent question that doesn't involve actual values by using smart numbers. Let's assign a smart number of 1000 to last year's computer budget (b) and a smart number 100 to last year's computer price (p). 1000 and 100 are easy numbers to take a percent of.

This year's budget will equal $1000 \times 1.6 = 1600$
This year's computer price will equal $100 \times 1.2 = 120$

Now we can calculate the number of computers purchased each year, $q = b/p$
Number of computers purchased last year = 1000/100 = 10

Number of computers purchased this year = 1600/120 = 13 1/3 (while 1/3 of a computer doesn't make sense it won't affect the calculation)

	p	q	b
This Year	100	10	1000
Last Year	120	13 1/3	1600

The question is asking for the percent increase in quantity from last year to this year =

$$\frac{\text{new} - \text{old}}{\text{old}} \times 100\% = \frac{13\ 1/3 - 10}{10} \times 100\% = 33\ 1/3\%$$

This question could also have been solved algebraically by converting the percent increases into fractions.

Last year: $pq = b$, so $q = b/p$
This year: $(6/5)(p)(x) = (8/5)b$
If we solve for x (this year's quantity), we get $x = (8/5)(5/6)b/p$ or $(4/3)b/p$
If this year's quantity is 4/3 of last year's quantity (b/p), this represents a 33 1/3% increase.

The correct answer is A.

Concept # 4: Average Speed: No formula here:

1. We begin by figuring out Lexy's average speed. On her way from A to B, she travels 5 miles in one hour, so her speed is 5 miles per hour. On her way back from B to A, she travels the same 5 miles at 15 miles per hour. Her average speed for the round trip is NOT simply the average of these two speeds. Rather, her average speed must be computed using the formula $RT = D$, where R is rate, T is time and D is distance. Her average speed for the **whole** trip is the **total** distance of her trip divided by the **total** time of her trip.

   We already know that she spends 1 hour going from A to B. When she returns from B to A, Lexy travels 5 miles at a rate of 15 miles per hour, so our formula tells us that $15T = 5$, or $T = 1/3$. In other words, it only takes Lexy 1/3 of an hour, or 20 minutes, to return from B to A. Her total distance traveled for the round trip is 5+5=10 miles and her total time is 1+1/3=4/3 of an hour, or 80 minutes.

   We have to give our final answer in minutes, so it makes sense to find Lexy's average rate in miles per minute, rather than miles per hour. 10 miles / 80 minutes = 1/8 miles per minute. This is Lexy's average rate.

   We are told that Ben's rate is half of Lexy's, so he must be traveling at 1/16 miles per minute. He also travels a total of 10 miles, so $(1/16)T = 10$, or $T = 160$. Ben's round trip takes 160 minutes.

Alternatively, we could use a shortcut for the last part of this problem. We know that Ben's rate is half of Lexy's average rate. This means that, for the entire trip, Ben will take twice as long as Lexy to travel the same distance. Once we determine that Lexy will take 80 minutes to complete the round trip, we can double the figure to get Ben's time. $80 \times 2 = 160$.

The correct answer is D.

2. There is an important key to answering this question correctly: this is not a simple average problem but a weighted average problem. A weighted average is one in which the different parts to be averaged are not equally balanced. One is "worth more" than the other and skews the "simple" average in one direction. In addition, we must note a unit change in this problem: we are given rates in miles per hour but asked to solve for rates in miles per minute.

Average rate uses the same D = RT formula we use for rate problems but we have to figure out the different lengths of time it takes Dan to run and swim along the total 4-mile route. Then we have to take the 4 miles and divide by that total time. First, Dan runs 2 miles at the rate of 10 miles per hour. 10 miles per hour is equivalent to 1 mile every 6 minutes, so Dan takes 12 minutes to run the 2 miles. Next, Dan swims 2 miles at the rate of 6 miles per hour. 6 miles per hour is equivalent to 1 mile every 10 minutes, so Dan takes 20 minutes to swim the two miles.

Dan's total time is 12 + 20 = 32 minutes. Dan's total distance is 4 miles. Distance / time = 4 miles / 32 minutes = 1/8 miles per minute.

Note that if you do not weight the averages but merely take a simple average, you will get 2/15, which corresponds to incorrect answer choice B. 6 mph and 10 mph average to 8mph. (8mph)(1h/60min) = 8/60 miles/minute or 2/15 miles per minute.

The correct answer is A.

Concept # 5: Speed and Distance: Speed = Distance / Time

3.

Distance = Rate × Time, or $D = RT$.

(1) INSUFFICIENT: This statement tells us Harry's rate, 30 mph. This is not enough to calculate the distance from his home to his office, since we don't know anything about the time required for his commute.

$D = RT = (30 \text{ mph})\ (T)$

D cannot be calculated because T is unknown.

(2) INSUFFICIENT: If Harry had traveled twice as fast, he would have gotten to work in half the time, which according to this statement would have saved him 15 minutes. Therefore, his actual commute took 30 minutes. So we learn his commute time from this

statement, but don't know anything about his actual speed.
$D = RT = (R)$ (1/2 hour)
D cannot be calculated because R is unknown.

(1) AND (2) SUFFICIENT: From statement (1) we learned that Harry's rate was 30 mph. From Statement (2) we learned that Harry's commute time was 30 minutes. Therefore, we can use the rate formula to determine the distance Harry traveled.
$D = RT =$ (30 mph) (1/2 hour) = 15 miles

The correct answer is C.

4. To determine Bill's average rate of movement, first recall that Rate × Time = Distance. We are given that the moving walkway is 300 feet long, so we need only determine the time elapsed during Bill's journey to determine his average rate.

   There are two ways to find the time of Bill's journey. First, we can break down Bill's journey into two legs: walking and standing. While walking, Bill moves at 6 feet per second. Because the walkway moves at 3 feet per second, Bill's foot speed along the walkway is 6 – 3 = 3 feet per second. Therefore, he covers the 120 feet between himself and the bottleneck in (120 feet)/(3 feet per second) = 40 seconds.

   Now, how far along is Bill when he stops walking? While that 40 seconds elapsed, the crowd would have moved (40 seconds)(3 feet per second) = 120 feet. Because the crowd already had a 120 foot head start, Bill catches up to them at 120 + 120 = 240 feet. The final 60 feet are covered at the rate of the moving walkway, 3 feet per second, and therefore require (60 feet)/(3 feet per second) = 20 seconds. The total journey requires 40 + 20 = 60 seconds, and Bill's rate of movement is (300 feet)/(60 seconds) = 5 feet per second.

   Short-cut: This problem may also be solved with a shortcut. Consider that Bill's journey will end when the crowd reaches the end of the walkway (as long as he catches up with the crowd before the walkway ends). When he steps on the walkway, the crowd is 180 feet from the end. The walkway travels this distance in (180 feet)/(3 feet per second) = 60 seconds, and Bill's average rate of movement is (300 feet)/(60 seconds) = 5 feet per second.

   The correct answer is E.

5. It is easier to break this motion up into different segments. Let's first consider the 40 minutes up until John stops to fix his flat.

   40 minutes is 2/3 of an hour.

In 2/3 of an hour, John traveled 15 × 2/3 = 10 miles ($rt = d$)
In that same 2/3 of an hour, Jacob traveled 12 × 2/3 = 8 miles
John therefore had a two-mile lead when he stopped to fix his tire.

It took John 1 hour to fix his tire, during which time Jacob traveled 12 miles. Since John began this 1-hour period 2 miles ahead, at the end of the period he is 12 – 2 = 10 miles behind Jacob.
The question now becomes "how long does it take John to bridge the 10-mile gap between him and Jacob, plus whatever additional distance Jacob has covered, while traveling at 15 miles per hour while Jacob is traveling at 12 miles per hour?" We can set up an $rt = d$ chart to solve this.

	John	Jacob
R	15	12
T	t	t
D	$d + 10$	d

John's travel during this "catch-up period" can be represented as $15t = d + 10$
Jacob's travel during this "catch-up period" can be represented as $12t = d$

If we solve these two simultaneous equations, we get:
$15t = 12t + 10$
$3t = 10$
$t = 3\ 1/3$ hours

Another way to approach this question is to note that when John begins to ride again, Jacob is 10 miles ahead. So John must make up those first 10 miles plus whatever additional distance Jacob has covered while both are riding. Since Jacob's additional distance at any given moment is $12t$ (measuring from the moment when John begins riding again) we can represent the distance that John has to make up as $12t + 10$. We can also represent John's distance at any given moment as $15t$. Therefore, $15t = 12t + 10$, when John catches up to Jacob. We can solve this question as outlined above.

The correct answer is B.

Concept # 7: Work:

1. Let a be the number of hours it takes Machine A to produce 1 widget on its own. Let b be the number of hours it takes Machine B to produce 1 widget on its own.

   The question tells us that Machines A and B together can produce 1 widget in 3 hours. Therefore, in 1 hour, the two machines can produce 1/3 of a widget. In 1 hour, Machine

A can produce $1/a$ widgets and Machine B can produce $1/b$ widgets. Together in 1 hour, they produce $1/a + 1/b = 1/3$ widgets.

If Machine A's speed were doubled it would take the two machines 2 hours to produce 1 widget. When one doubles the speed, one cuts the amount of time it takes in half. Therefore, the amount of time it would take Machine A to produce 1 widget would be $a/2$. Under these new conditions, in 1 hour Machine A and B could produce $1/(a/2) + 1/b = 1/2$ widgets. We now have two unknowns and two different equations. We can solve for a.

The two equations:
$2/a + 1/b = 1/2$ (Remember, $1/(a/2) = 2/a$)
$1/a + 1/b = 1/3$

Subtract the bottom equation from the top:
$2/a – 1/a = 1/2 – 1/3$
$1/a = 3/6 – 2/6$
$1/a = 1/6$
Therefore, $a = 6$.

The correct answer is E.

2. Tom's individual rate is 1 job / 6 hours or 1/6.
During the hour that Tom works alone, he completes 1/6 of the job (using $rt = w$).

Peter's individual rate is 1 job / 3 hours.
Peter joins Tom and they work together for another hour; Peter and Tom's respective individual rates can be added together to calculate their combined rate: 1/6 + 1/3 = 1/2.
Working together then they will complete 1/2 of the job in the 1 hour they work together.

At this point, 2/3 of the job has been completed (1/6 by Peter alone + 1/2 by Peter and Tom), and 1/3 remains.

When John joins Tom and Peter, the new combined rate for all three is: 1/6 + 1/3 + 1/2 = 1.
The time that it will take them to finish the remaining 1/3 of the job can be solved:
$rt = w \longrightarrow (1)(t) = 1/3 \longrightarrow t = 1/3$.

The question asks us for the fraction of the job that Peter completed. In the hour that Peter worked with Tom he alone completed: $rt = w \longrightarrow w = (1/3)(1) = 1/3$ of the job.
In the last 1/3 of an hour that all three worked together, Peter alone completed: (1/3)(1/3) = 1/9 of the job.
Adding these two values together, we get 1/3 + 1/9 of the job = 4/9 of the job.

The correct answer is E.

3.
To find the combined rate of Machines A and B, we combine their individual rates. If Machine A can fill an order of widgets in *a* hours, then in 1 hour it can fill $\frac{1}{a}$ of the order. By the same token, if Machine B can fill the order of widgets in *b* hours, then in 1 hour, it can fill $\frac{1}{b}$ of the order. So together in 1 hour, Machines A and B can fill $\frac{1}{a}+\frac{1}{b}$ of the order:

$$\frac{1}{a}+\frac{1}{b}=\frac{(b)1}{(b)(a)}+\frac{(a)1}{(a)(b)}=\frac{b}{ab}+\frac{a}{ab}=\frac{a+b}{ab}$$

So in 1 hour, Machines A and B can complete $\frac{a+b}{ab}$ of the order. To find the number of hours the machines need to complete the *entire* order, we can set up the following equation:

(fraction of order completed in 1 hour) x (number of hours needed to complete entire order) = 1 order.

If we substitute $\frac{a+b}{ab}$ for the fraction of the order completed in 1 hour, we get:

$\frac{a+b}{ab}(x)=1$, where *x* is the number of hours needed to complete the entire order. If we divide both sides by $\frac{a+b}{ab}$, we get:

$$x=\frac{ab}{a+b}$$

In other words, it will take Machines A and B $\frac{ab}{a+b}$ hours to complete the entire order working together at their respective rates.

The question stem tells us that *a* and *b* are both even integers. We are then asked whether *a* and *b* are equal. If they are equal, we can express each as 2*z*, where *z* is a non-zero integer, because they are even. If we replace *a* and *b* with 2*z* in the combined rate, we get:

$$\frac{(2z)(2z)}{2z+2z}=\frac{4z^2}{4z}=z$$

So if *a* and *b* are equal, the combined rate of Machines A and B must be an integer (since *z* is an integer). We can rephrase the question as:

Is the combined rate of Machines A and B an integer?

Statement 1 tells us that it took 4 hours and 48 minutes for the two machines to fill the order (remember, they began at noon). This shows that the combined rate of Machines A and B is NOT an integer (otherwise, it would have taken the machines a whole number of hours to complete the order). So we know that *a* and *b* cannot be the same. Sufficient.

Statement 2 tells us that $(a + b)^2 = 400$. Since both *a* and *b* must be positive (because they represent a number of hours), we can take the square root of both sides of the equation without having to worry about negative roots. Therefore, it must be true that $a + b = 20$. So it is possible that $a = 10$ and that $b = 10$, which would allow us to answer "yes" to the question. But it is also possible that $a = 12$ and $b = 8$ (or any other combination of positive even integers that sum to 20), which would give us a "no". Insufficient.

The correct answer is A: Statement 1 alone is sufficient, but statement 2 alone is not.

Concept # 8: Compound Interest formula: $\boldsymbol{A = P\left(1 + \frac{R}{100}\right)^n}$ CI = A – P.

1.
Compound interest is computed using the following formula:

$F = P\ (\ 1 + r/n)^{nt}$, where
F = Final value
P = Principal
r = annual interest rate
n = number of compounding periods per year
t = number of years

From the question, we can deduce the following information about the growth during this period:

At the end of the *x* years, the final value, *F*, will be equal to 16 times the principal (the money is growing by a factor of 16).
Therefore, $F = 16P$.
$r = .08$ (8% annual interest rate)
$n = 4$ (compounded quarterly)
$t = x$ (the question is asking us to express the time in terms of *x* number of years)

We can write the equation
$16P = P\ (1 + .08/4)^{4x}$
$16 = (1.02)^{4x}$

Now we can take the fourth root of both sides of the equation. (i.e.the equivalent of taking the square root twice) We will only consider the positive root because a negative 2 doesn't

make sense here.
$16^{1/4} = [(1.02)^{4x}]^{1/4}$
$2 = (1.02)^x$

The correct answer is B.

2. We need to consider the formula for compound interest for this problem: $F = P(1 + r)^x$, where F is the final value of the investment, P is the principal, r is the interest rate per compounding period as a decimal, and x is the number of compounding periods (NOTE: sometimes the formula is written in terms of the annual interest rate, the number of compounding periods per year and the number of years). Let's start by manipulating the given expression for r:

$$r = 100\left(\sqrt{\frac{v+q}{p}} - 1\right) \rightarrow \frac{r}{100} = \sqrt{\frac{v+q}{p}} - 1 \rightarrow 1 + \frac{r}{100} = \sqrt{\frac{v+q}{p}} \rightarrow$$

$$\left(1 + \frac{r}{100}\right)^2 = \left(\sqrt{\frac{v+q}{p}}\right)^2 \rightarrow \left(1 + \frac{r}{100}\right)^2 = \frac{v+q}{p} \rightarrow p\left(1 + \frac{r}{100}\right)^2 = v + q \rightarrow$$

$$v = p\left(1 + \frac{r}{100}\right)^2 - q$$

Let's compare this simplified equation to the compound interest formula. Notice that r in this simplified equation (and in the question) is not the same as the r in the compound interest formula. In the formula, the r is already expressed as a decimal equivalent of a percent, in the question the interest is r percent. The simplified equation, however, deals with this discrepancy by dividing r by 100.

In our simplified equation, the cost of the share of stock (p), corresponds to the principal (P) in the formula, and the final share price (v) corresponds to the final value (F) in the formula. Notice also that the exponent 2 corresponds to the x in the formula, which is the number of compounding periods. By comparing the simplified equation to the compound interest formula, we see that the equation tells us that the share rose at the daily interest rate of p percent for TWO days. Then the share lost a value of q dollars on the third day, i.e. the "– q" portion of the expression. If the investor bought the share on Monday, she sold it three days later on Thursday.

The correct answer is B.

Concept # 9: Population Growth:

1.

To solve a population growth question, we can use a population chart to track the growth. The annual growth rate in this question is unknown, so we will represent it as x. For example, if the population doubles each year, $x = 2$; if it grows by 50% each year, $x = 1.5$. Each year the population is multiplied by this factor of x.

Time	Population
Now	500

in 1 year	$500x$
in 2 years	$500x^2$
:	:
in n years	$500x^n$

The question is asking us to find the minimum number of years it will take for the herd to double in number. In other words, we need to find the minimum value of n that would yield a population of 1000 or more.

We can represent this as an inequality:
$500x^n > 1000$
$x^n > 2$

In other words, we need to find what integer value of n would cause x^n to be greater than 2. To solve this, we need to know the value of x. Therefore, we can rephrase this question as: "What is x, the annual growth factor of the herd?"

(1) INSUFFICIENT: This tells us that in ten years the following inequality will hold:
$500x^{10} > 5000$
$x^{10} > 10$

There are an infinite number of growth factors, x, that satisfy this inequality.
For example, $x = 1.5$ and $x = 2$ both satisfy this inequality.
If $x = 2$, the herd of antelope doubles after one year.
If $x = 1.5$, the herd of antelope will be more than double after two years 500(1.5)(1.5) = 500(2.25).

(2) SUFFICIENT: This will allow us to find the growth factor of the herd. We can represent the growth factor from the statement as y. (NOTE y does not necessarily equal $2x$ because x is a growth factor. For example, if the herd actually grows at a rate of 10% each year, $x = 1.1$, but $y = 1.2$, i.e. 20%)

Time	Population
Now	500
in 1 year	$500y$
in 2 years	$500y^2$

According to the statement, $500y^2 = 980$
$y^2 = 980/500$
$y^2 = 49/25$
$y = 7/5$ OR 1.4 (y can't be negative because we know the herd is growing)
This means that the hypothetical double rate from the statement represents an annual growth rate of 40%.
The actual growth rate is therefore 20%, so $x = 1.2$.

The correct answer is B.

2.

Let's say:
I = the original amount of bacteria
F = the final amount of bacteria
t = the time bacteria grows
If the bacteria increase by a factor of x every y minutes, we can represent the growth of the bacteria with the equation:
$F = I(x)^{t/y}$

To understand why, let's assign some values to I, x and y:

I =	100
x =	2
y =	3

If the bacteria start off 100 in number and they double every 3 minutes, after 3 minutes there will be 100(2) bacteria. Let's construct a table to track the growth of the bacteria:

t (time)	F (final count)
3	$100(2) = 100(2)^1$
6	$100(2)(2) = 100(2)^2$
9	100(2)(2)(2) = 100(2)3
12	$100(2)(2)(2)(2) = 100(2)^4$

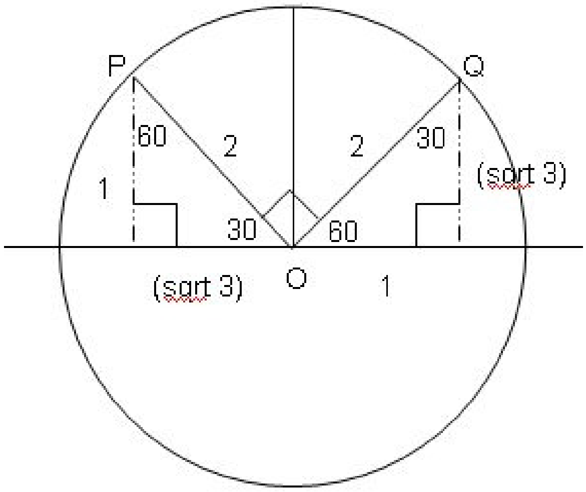
We can generalize the F values in the table as $100(2)^n$.
The 100 represents the initial count, I.
The 2 represents the factor of growth (in this problem x).
The n represents the number of growth periods. The number of growth periods is found by dividing the time, t, by the amount of time it takes to complete a period, y.
From this example, we can extrapolate the general formula for exponential growth: $F = I(x)^{t/y}$
This question asks us how long it will take for the bacteria to grow to 10,000 times their original amount.
The bacteria will have grown to 10,000 times their original amount when $F = 10{,}000I$.
If we plug this into the general formula for exponential growth, we get: $10{,}000I= I(x)^{t/y}$ or $10{,}000 = (x)^{t/y}$.
The question is asking us to solve for t.
(1) SUFFICIENT: This statement tells us that $x^{1/y}=10$. If we plug this value into the equation we can solve for t.
$10{,}000 = (x)^{t/y}$
$10{,}000 =[(x)^{1/y}]^t$
$10{,}000 =(10)^t$
$t = 4$
(2) SUFFICIENT: The bacteria grow one hundredfold in 2 minutes, that is to say they grow by a

factor of 10^2. Since exponential growth is characterize by a constant factor of growth (i.e. by x every y minutes), for the bacteria to grow 10,000 fold (i.e. a factor of 10^4), they will need to grow another 2 minutes, for a total of four minutes ($10^2 \times 10^2 = 10^4$).
The correct answer is D, EACH statement ALONE is sufficient to answer the question.

Concept # 10: 30-60-90 Right angled triangle:

1.

First, see that after dropping perpendicular lines, we have two right triangles.



Let's begin with the triangle on the left.

We know the sides are 1 and (sqrt 3) from point P.
If you know your special right triangles, you will quickly see that this is a 30-60-90 right triangle.

The angle opposite '1' is 30 degrees.

Let's move on to the triangle on the right.

We know that a straight line has 180 degrees.

Since we know the lower angle of the triangle on the left is 30 degrees, and we also know the angle between the two line segments is 90 degrees, the lower angle of the triangle on the right must be 60 degrees in order to sum to 180 degrees. (30 + 90 + x = 180; x = 60)

This means the triangle on the right is also a 30-60-90 triangle. The hypotenuse of this triangle is the same as the other triangle's (which is '2' by the Pythagorean Theorem), since both are radii of the same circle.

Using the same properties of a 30-60-90 triangle, you can find the side lengths and finally the point (s,t) which gives the value for s = 1.

2.
The formula for the area of a triangle is 1/2(bh). We know the height of ΔABC. In order to solve for area, we need to find the length of the base. We can rephrase the question:

What is BC?

(1) INSUFFICIENT: If angle ABD = 60°, ΔABD must be a 30-60-90 triangle. Since the proportions of a 30-60-90 triangle are x: x $\sqrt{3}$: 2x (shorter leg: longer leg: hypotenuse), and AD = 6 $\sqrt{3}$, BD must be 6. We know nothing about DC.

(2) INSUFFICIENT: Knowing that AD = 6 $\sqrt{3}$, and AC = 12, we can solve for CD by recognizing that ΔACD must be a 30-60-90 triangle (since it is a right triangle and two of its sides fit the 30-60-90 ratio), or by using the Pythagorean theorem. In either case, CD = 6, but we know nothing about BD.

(1) AND (2) SUFFICIENT: If BD = 6, and DC = 6, then BC = 12, and
the area of ΔABC = 1/2(bh) = 1/2(12)(6 $\sqrt{3}$) = 36 $\sqrt{3}$.

The correct answer is C

3.
Triangle *DBC* is inscribed in a semicircle (that is, the hypotenuse CD is a diameter of the circle). Therefore, angle *DBC* must be a right angle and triangle *DBC* must be a right triangle.

(1) SUFFICIENT: If the length of *CD* is twice that of *BD*, then the ratio of the length of *BD* to the length of the hypotenuse *CD* is 1 : 2. Knowing that the side ratios of a 30-60-90 triangle are 1 : $\sqrt{3}$: 2, where 1 represents the short leg, $\sqrt{3}$ represents the long leg, and 2 represents the hypotenuse, we can conclude that triangle *DBC* is a 30-60-90 triangle. Since side *BD* is the short leg, angle *x*, the angle opposite the short leg, must be the smallest angle (30 degrees).

(2) SUFFICIENT: If triangle *DBC* is inscribed in a semicircle, it must be a right triangle. So, angle *DBC* is 90 degrees. If *y* = 60, *x* = 180 – 90 – 60 = 30.

The correct answer is D.

4. In order to find the area of the triangle, we need to find the lengths of a base and its associated height. Our strategy will be to prove that ABC is a right triangle, so that CB will be the base and AC will be its associated height.

(1) INSUFFICIENT: We now know one of the angles of triangle ABC, but this does not provide sufficient information to solve for the missing side lengths.

(2) INSUFFICIENT: Statement (2) says thπ the circumference of the circle is 18 π. Since the circumference of a circle equals times the diameter, the diameter of the circle is 18. Therefore AB is a diameter. However, point C is still free to "slide" around the circumference of the circle giving different areas for the triangle, so this is still insufficient to solve for the area of the triangle.

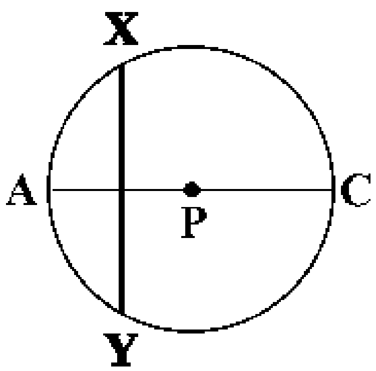
(1) AND (2) SUFFICIENT: Note that inscribed triangles with one side on the diameter of the circle must be right triangles. Because the length of the diameter indicated by Statement (2) indicates that segment AB equals the diameter, triangle ABC must be a right triangle. Now, given Statement (1) we recognize that this is a 30-60-90 degree triangle. Such triangles always have side length ratios of

$1:\sqrt{3}:2$

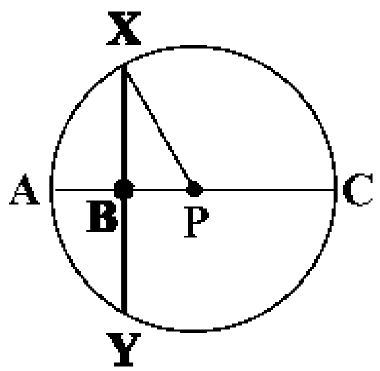
Given a hypotenuse of 18, the other two segments AC and CB must equal 9 and $9\sqrt{3}$ respectively. This gives us the base and height lengths needed to calculate the area of the triangle, so this is sufficient to solve the problem.

The correct answer is C.

5.
Since no picture is given in the problem, draw it. Below, find the given circle with center *P* and cord *XY* bisecting radius *AP*.



Although, the picture above is helpful, drawing in an additional radius is often an important step towards seeing the solution. Thus, we will add to the picture by drawing in radius *XP* as shown below.



Since *XY* bisects radius *AP* at point *B*, segment *BP* is half the length of any radius.

Since *BP* is half the length of radius XP, right triangle *XBP* must be a 30-60-90 triangle with sides in the ratio of $1 : \sqrt{3} : 2$.

Therefore, finding the length of any side of the triangle, will give us the lengths of the other two sides.

Finding the length of radius *XP* will give us the length of *XB*, which is half the length of cord *XY*. Thus, in order to answer the question--**what is the length of cord *XY*?**--we need to know only one piece of information:

The length of the radius of the circle.

Statement (1) alone tells us that the circumference of the circle is twice the area of the circle. Using this information, we can set up an equation and solve for the radius as follows:

$$\text{Circum} = 2 \times \text{Area}$$
$$2\pi r = 2(\pi r^2)$$
$$2r = 2r^2$$
$$r = r^2$$
$$r = 1$$

Therefore Statement (1) alone is sufficient to answer the question.

Statement (2) alone tells us that the length of Arc *XAY* = $\frac{2\pi}{3}$.

Arc *XAY* is made up of arc *XA* + arc *AY*.

Given that triangle *XBP* is a 30:60: 90 triangle, we know that <*XPA* = 60 degrees and can deduce that <*APY* = 60 degrees as well. Therefore Arc *XAY* =120 degrees or 1/3 of the circumference of the circle. Using this information, we can solve for the radius of the circle by setting up an equation as follows:

$$\text{Arc } XAY = \frac{1}{3}\text{Circum}$$

$$\frac{2\pi}{3} = \frac{1}{3}(2\pi r)$$

$$2\pi = 2\pi r$$

$$1 = r$$

Therefore, Statement (2) alone is also sufficient to answer the question. The correct answer is D, each statement ALONE is sufficient.

Concept # 11: 45-45-90 Right angled triangle (isosceles right triangle):

1.
(1) INSUFFICIENT: This tells us that *AC* is the height of triangle *BAD* to base *BD*. This does not help us find the length of *BD*.
(2) INSUFFICIENT: This tells us that *C* is the midpoint of segment *BD*. This does not help us find the length of *BD*.
(1) AND (2) SUFFICIENT: Using statements 1 and 2, we know that *AC* is the perpendicular bisector of *BD*. This means that triangle *BAD* is an isosceles triangle so side AB must have a length of 5 (the same length as side *AD*). We also know that angle *BAD* is a right angle, so side *BD* is the hypotenuse of right isosceles triangle *BAD*. If each leg of the triangle is 5, the hypotenuse (using the Pythagorean theorem) must be 5 $\sqrt{2}$.

The correct answer is C.

2.
The question stem tells us that ABCD is a rectangle, which means that triangle ABE is a right triangle.

The formula for the area of any triangle is: 1/2 (Base X Height).

In right triangle ABE, let's call the base AB and the height BE. Thus, we can rephrase the questions as follows: **Is 1/2 (AB X BE) greater than 25?**

Let's begin by analyzing the first statement, taken by itself. Statement (1) tells us that the length of AB = 6. While this is helpful, it provides no information about the length of BE. Therefore there is no way to determine whether the area of the triangle is greater than 25 or not.

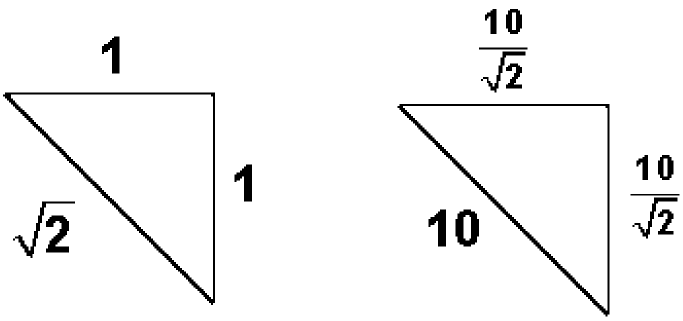
Now let's analyze the second statement, taken by itself. Statement (2) tells us that length of diagonal AE = 10. We may be tempted to conclude that, like the first statement, this does not give us the two pieces of information we need to know (that is, the lengths of AB and BE respectively). However, knowing the length of the diagonal of the right triangle actually does provide us with some very relevant information about the lengths of the base (AB) and the height (BE).

Consider this fact: Given the length of the diagonal of a right triangle, it **IS** possible to determine the maximum area of that triangle.

How? **The right triangle with the largest area will be an isosceles right triangle (where both the base and height are of equal length).**

Therefore, given the length of diagonal AE = 10, we can determine the largest possible area of triangle ABE by making it an isosceles right triangle.

That important side ratio is $1 : 1 : \sqrt{2}$ where the two **1**'s represent the two legs (the base and the height) and $\sqrt{2}$ represents the diagonal. Thus if we are to construct an isosceles right triangle with a diagonal of 10, then, using the side ratios, we can determine that each leg will have a length of $\frac{10}{\sqrt{2}}$.



Now, we can calculate the area of this isosceles right triangle:

$$\frac{1}{2}(AB \times BE) = \frac{1}{2}\left(\frac{10}{\sqrt{2}} \times \frac{10}{\sqrt{2}}\right) = \frac{1}{2}\left(\frac{100}{2}\right) = \frac{1}{2}(50) = 25$$

Since an isosceles right triangle will yield the maximum possible area, we know that 25 is the maximum possible area of a right triangle with a diagonal of length 10.

Of course, we don't really know if 25 is, in fact. the area of triangle ABE, but we do know that 25 is the maximum possible area of triangle ABE. Therefore we are able to answer our original question: Is the area of triangle ABE greater than 25? *NO it is not greater than 25, because the maximum area is 25.*

Since we can answer the question using Statement (2) alone, the correct answer is B.

3.
Let the hypotenuse be x, then the length of the leg is x/root2.

x+2x/root2=16+16*root2
x+root*x=16+16*root2
So, x=16

Concept # 12: Right angled triangle

Concept # 13: Similar Triangles:

1.

USE h^2 = mn, so 4^2 = 3 * *x* so *x* = 16/3

OR

Because angles *BAD* and *ACD* are right angles, the figure above is composed of three *similar* right triangles: *BAD*, *ACD* and *BCA*. [Any time a height is dropped from the right angle vertex of a right triangle to the opposite side of that right triangle, the three triangles that result have the same 3 angle measures. This means that they are similar triangles.] To solve for the length of side *CD*, we can set up a proportion, based on the relationship between the similar triangles *ACD* and *BCA*: BC/AC = CA/CD or ¾ = 4/CD or CD = 16/3. The correct answer is D.

2.
Use A1 / A2 = $(L1 / L2)^2$ So we have $1/12 = (3/(3+x))^2$ or $1/\sqrt{12} = 3/(3+x)$ or $1/2\sqrt{3} = 3/(3+x)$ so $x = 6\sqrt{3} - 3$.

3.
Since *BE* ∥ *CD*, triangle *ABE* is similar to triangle *ACD* (parallel lines imply two sets of equal angles). We can use this relationship to set up a ratio of the respective sides of the two triangles:

$$\frac{AB}{AC} = \frac{AE}{AD}$$

$$\frac{3}{6} = \frac{4}{AD}$$ So *AD* = 8.

We can find the area of the trapezoid by finding the area of triangle *CAD* and subtracting the area of triangle *ABE*.

Triangle *CAD* is a right triangle since it has side lengths of 6, 8 and 10, which means that triangle *BAE* is also a right triangle (they share the same right angle).
Area of trapezoid = area of triangle *CAD* – area of triangle *BAE*
= (1/2)*bh* – (1/2)*bh*
= 0.5(6)(8) – 0.5(3)(4)

= 24 – 6
= 18
The correct answer is B

4.

For GMAT triangle problems, one useful tool is the similar triangle strategy. Triangles are defined as similar if all their corresponding angles are equal or if the lengths of their corresponding sides have the same ratios.

(1) INSUFFICIENT: Just knowing that $x = 60°$ tells us nothing about triangle *EDB*. To illustrate, note that the exact location of point *E* is still unknown. Point *E* could be very close to the circle, making *DE* relatively short in length. However, point *E* could be quite far away from the circle, making *DE* relatively long in length. We cannot determine the length of *DE* with certainty.

(2) SUFFICIENT: If *DE* is parallel to *CA*, then (angle *EDB*) = (angle *ACB*) = x. Triangles *EBD* and *ABC* also share the angle *ABC*, which of course has the same measurement in each triangle. Thus, triangles *EBD* and *ABC* have two angles with identical measurements. Once you find that triangles have 2 equal angles, you know that the third angle in the two triangles must also be equal, since the sum of the angles in a triangle is 180°.

So, triangles *EBD* and *ABC* are similar. This means that their corresponding sides must be in proportion:

$CB/DB = AC/DE$
radius/diameter = radius/DE
$3.5/7 = 3.5/DE$

Therefore, DE = diameter = 7.

The correct answer is B.

5.

First, recall that in a right triangle, the two shorter sides intersect at the right angle. Therefore, one of these sides can be viewed as the base, and the other as the height. Consequently, the area of a right triangle can be expressed as one half of the product of the two shorter sides (i.e., the same as one half of the product of the height times the base). Also, since *AB* is the hypotenuse of triangle *ABC*, we know that the two shorter sides are *BC* and *AC* and the area of triangle $ABC = (BC \times AC)/2$. Following the same logic, the area of triangle $KLM = (LM \times KM)/2$.

Also, the area of *ABC* is 4 times greater than the area of *KLM*:
$(BC \times AC)/2 = 4(LM \times KM)/2$
$BC \times AC = 4(LM \times KM)$

(1) SUFFICIENT: Since angle *ABC* is equal to angle *KLM*, and since both triangles have a right angle, we can conclude that the angles of triangle *ABC* are equal to the angles of triangle *KLM*, respectively (note that the third angle in each triangle will be equal to 35 degrees, i.e., 180 – 90 – 55 = 35). Therefore, we can conclude that triangles *ABC* and *KLM* are similar. Consequently, the respective sides of these triangles will be proportional, i.e. $AB/KL = BC/LM = AC/KM = x$, where x is the coefficient of proportionality (e.g., if *AB* is twice as long as *KL*, then $AB/KL = 2$ and for every side in triangle KLM, you could multiply that side by 2 to get the corresponding side in triangle ABC).

We also know from the problem stem that the area of *ABC* is 4 times greater than the area of *KLM*, yielding $BC \times AC = 4(LM \times KM)$, as discussed above.

Knowing that $BC/LM = AC/KM = x$, we can solve the above expression for the coefficient of proportionality, x, by plugging in $BC= x(LM)$ and $AC = x(KM)$:
$BC \times AC = 4(LM \times KM)$
$x(LM) \times x(KM) = 4(LM \times KM)$
$x^2 = 4$
$x = 2$ (since the coefficient of proportionality cannot be negative)

Thus, we know that $AB/KL = BC/LM = AC/KM = 2$. Therefore, $AB = 2KL = 2(10) = 20$

(2) INSUFFICIENT: This statement tells us the length of one of the shorter sides of the triangle *KLM*. We can compute all the sides of this triangle (note that this is a 6-8-10 triangle) and find its area (i.e., (0.5)(6)(8) = 24); finally, we can also calculate that the area of the triangle *ABC* is equal to 96 (four times the area of *KLM*). We determined in the first paragraph of the explanation, above, that the area of $ABC = (BC \times AC)/2$. Therefore: $96 = (BC \times AC)/2$ and $192 = BC \times AC$. We also know the Pythagorean theorem: $(BC)^2 + (AC)^2= (AB)^2$. But there is no way to convert $BC \times AC$ into $(BC)^2 + (AC)^2$ so we cannot determine the hypotenuse of triangle ABC.

The correct answer is A.

6.

THIS IS AN 800 LEVEL PROBLEM.

Innovative approach: Imagine right angled triangle with sides 12.
As the smallest right triangle is 3, 4, 5… and 12 is a multiple of 3 and 4 both, we may make

3, 4, 5 MULTIPLIED by 3 = 9, 12, 15… and 3, 4, 5 MULTIPLIED by 4 = 12, 16, 20.

Now let's check the perimeter. 16 + 9 + 20 + 15 = 60, so these are the correct combinations.
Area ratio = $(L1/L2)^2 = (4/3)^2 = 16/9$.

Short-cut: The area of two similar figures will be in the ratio of squares of sides. So the best guess answer is 16/9.

Detailed Solution:

We are given a right triangle PQR with perimeter 60 and a height to the hypotenuse QS of length 12. We're asked to find the ratio of the area of the larger internal triangle PQS to the area of the smaller internal triangle RQS.

First let's find the side lengths of the original triangle. Let c equal the length of the hypotenuse PR, and let a and b equal the lengths of the sides PQ and QR respectively. First of all we know that:

(1) $a^2 + b^2 = c^2$ Pythagorean Theorem for right triangle PQR

(2) $ab/2 = 12c/2$ Triangle PQR's area computed using the standard formula (1/2*b*h) but using a different base-height combination:

- We can use base = leg a and height = leg b to get Area of PQR = ab/2
- We can also use base = hypotenuse c and height = 12 (given) to get Area of PQR = 12c/2
- The area of PQR is the same in both cases, so I can set the two equal to each other: ab/2 = 12c/2.

(3) $a + b + c = 60$ The problem states that triangle PQR's perimeter is 60

(4) $a > b$ PQ > QR is given

(5) $(a + b)^2 = (a^2 + b^2) + 2ab$ Expansion of $(a + b)^2$

(6) $(a + b)^2 = c^2 + 24c$ Substitute (1) and (2) into right side of (5)

(7) $(60 – c)^2 = c^2 + 24c$ Substitute $(a + b) = 60 – c$ from (3)

(8) $3600 – 120c + c^2 = c^2 + 24c$

(9) $3600 = 144c$

(10) $25 = c$

Substituting $c = 25$ into equations (2) and (3) gives us:

(11) $ab = 300$

(12) $a + b = 35$

which can be combined into a quadratic equation and solved to yield $a = 20$ and $b = 15$. The other possible solution of the quadratic is $a = 15$ and $b = 20$, which does not fit the requirement that $a > b$.

Remembering that a height to the hypotenuse always divides a right triangle into two smaller triangles that are similar to the original one (since they all have a right angle and they share another of the included angles), therefore all three triangles are similar to each other. Therefore their areas will be in the ratio of the square of their respective side lengths. The larger internal triangle has a hypotenuse of 20 (= *a*) and the smaller has a hypotenuse of 15 (= *b*), so the side lengths are in the ratio of 20/15 = 4/3. You must square this to get the ratio of their areas, which is $(4/3)^2 = 16/9$.

The correct answer is D.

7.

We are given a right triangle that is cut into four smaller right triangles. Each smaller triangle was formed by drawing a perpendicular from the right angle of a larger triangle to that larger triangle's hypotenuse. When a right triangle is divided in this way, two similar triangles are created. And each one of these smaller similar triangles is also similar to the larger triangle from which it was formed.

Thus, for example, triangle *ABD* is similar to triangle *BDC*, and both of these are similar to triangle *ABC*. Moreover, triangle *BDE* is similar to triangle *DEC*, and each of these is similar to triangle *BDC*, from which they were formed. If *BDE* is similar to *BDC* and *BDC* is similar to *ABD*, then *BDE* must be similar to *ABD* as well.

Remember that similar triangles have the same interior angles and the ratio of their side lengths are the same. So the ratio of the side lengths of *BDE* must be the same as the ratio of the side lengths of *ABD*. We are given the hypotenuse of *BDE*, which is also a leg of triangle *ABD*. If we had even one more side of *BDE*, we would be able to find the side lengths of *BDE* and thus know the ratios, which we could use to determine the sides of *ABD*.

(1) SUFFICIENT: If *BE* = 3, then *BDE* is a 3-4-5 right triangle. *BDE* and *ABD* are similar triangles, as discussed above, so their side measurements have the same proportion. Knowing the three side measurements of *BDE* and one of the side measurements of *ABD* is enough to allow us to calculate *AB*.

To illustrate:
BD = 5 is the hypotenuse of *BDE*, while *AB* is the hypotenuse of *ABD*.
The longer leg of right triangle *BDE* is *DE* = 4, and the corresponding leg in *ABD* is *BD* = 5.

Since they are similar triangles, the ratio of the longer leg to the hypotenuse should be the same in both *BDE* and *ABD*.
For *BDE*, the ratio of the longer leg to the hypotenuse = 4/5.
For *ABD*, the ratio of the longer leg to the hypotenuse = 5/*AB*.

Thus, 4/5 = 5/*AB*, or *AB* = 25/4 = 6.25

(2) SUFFICIENT: If *DE* = 4, then *BDE* is a 3-4-5 right triangle. This statement provides identical information to that given in statement (1) and is sufficient for the reasons given above.

The correct answer is D.

8.
In SIMILAR FIGURES, the RATIO OF AREAS is (RATIO OF LENGTHS)^2

In SIMILAR SOLIDS, the RATIO OF VOLUMES is (RATIO OF LENGTHS)^3
In SIMILAR SOLIDS, the RATIO OF SURFACE AREAS is (RATIO OF LENGTHS)^2
So in **similar figures: if length ratio = a : b, then area ratio = $a^2 : b^2$**
In **similar 3-d solids: length ratio = a : b, surface area ratio = $a^2 : b^2$, volume ratio = $a^3 : b^3$**

In this problem, you have a^2 : b^2 = 2 : 1. If you know the result(s) above, then it follows at once that a : b (the ratio of lengths, which is what you're looking for) is
√2 : 1. Ans. C.

Concept # 14: Lines and Angles:

1.
(1) INSUFFICIENT: We don't know any of the angle measurements.

(2) INSUFFICIENT: We don't know the relationship of *x* to *y*.

(1) AND (2) INSUFFICIENT: Because *l*1 is parallel to *l*2, we know the relationship of the four angles at the intersection of *l*2 and *l*3 (*l*3 is a transversal cutting two parallel lines) and the same four angles at the intersection of *l*1 and *l*3. We do not, however, know the relationship of *y* to those angles because we do not know if *l*3 is parallel to *l*4.

The correct answer is E.

2.

We are given two triangles and asked to determine the degree measure of *z*, an angle in one of them.

The first step in this problem is to analyze the information provided in the question stem. We are told that $x - q = s - y$. We can rearrange this equation to yield $x + y = s + q$. Since $x + y + z = 180$ and since $q + s + r = 180$, it must be true that $z = r$. We can now look at the statements.

Statement (1) tells us that $xq + sy + sx + yq = zr$. In order to analyze this equation, we need to rearrange it to facilitate factorization by grouping like terms: $xq + yq + sx + sy = zr$. Now we can factor:

$$xq + yq + sx + sy = zr \rightarrow$$
$$q(x + y) + s(x + y) = zr \rightarrow$$
$$(x + y)(q + s) = zr$$

Since $x + y = q + s$ and $z = r$, we can substitute and simplify:

$$(x + y)(q + s) = zr \rightarrow$$
$$(x + y)(x + y) = (z)(z) \rightarrow$$
$$\sqrt{(x + y)^2} = \sqrt{z^2} \rightarrow$$
$$x + y = z$$

Is this sufficient to tell us the value of z? Yes. Why? Consider what happens when we substitute z for $x + y$:

$$x + y + z = 180 \rightarrow$$
$$z + z = 180 \rightarrow$$
$$2z = 180 \rightarrow$$
$$z = 90$$

It is useful to remember that when the sum of two angles of a triangle is equal to the third angle, the triangle must be a right triangle. Statement (1) is sufficient.

Statement (2) tells us that $zq - ry = rx - zs$. In order to analyze this equation, we need to rearrange it:

$$zq - ry = rx - zs \rightarrow$$
$$zq + zs = rx + ry \rightarrow$$
$$z(q + s) = r(x + y) \rightarrow$$
$$z = \frac{r(x + y)}{(q + s)} \rightarrow$$
$$\frac{z}{r} = \frac{x + y}{q + s}$$

Is this sufficient to tell us the value of z? No. Why not? Even though we know the following:

$z = r$
$x + y = q + s$
$x + y + z = 180$
$q + r + s = 180$

we can find different values that will satisfy the equation we derived from statement (2):

$$\frac{90}{90} = \frac{30 + 60}{40 + 50}$$

or

$$\frac{100}{100} = \frac{40 + 40}{10 + 70}$$

These are just two examples. We could find many more. Since we cannot determine the value of z, statement (2) is insufficient.

The correct answer is A: Statement (1) alone is sufficient, but statement (2) is not.

3. The question asks us to find the degree measure of angle a. Note that a and e are equal since they are vertical angles, so it's also sufficient to find e.

Likewise, you should notice that $e + f + g = 180$ degrees. Thus, to find e, it is sufficient to find $f + g$. The question can be rephrased to the following: "What is the value of $f + g$?"

(1) SUFFICIENT: Statement (1) tells us that $b + c = 287$ degrees. This information allows us to calculate $f + g$. More specifically:

$b + c = 287$
$(b + f) + (c + g) = 180 + 180$ Two pairs of supplementary angles.
$b + c + f + g = 360$
$287 + f + g = 360$
$f + g = 73$

(2) INSUFFICIENT: Statement (2) tells us that $d + e = 269$ degrees. Since $e = a$, this is equivalent to $d + a = 269$. There are many combinations of d and a that satisfy this constraint, so we cannot determine a unique value for a.

The correct answer is A.

4.

The perimeter of a triangle is equal to the sum of the three sides.

(1) INSUFFICIENT: Knowing the length of one side of the triangle is not enough to find the sum of all three sides.

(2) INSUFFICIENT: Knowing the length of one side of the triangle is not enough to find the sum of all three sides.

Together, the two statements are SUFFICIENT. Triangle ABC is an isosceles triangle which means that there are theoretically 2 possible scenarios for the lengths of the three sides of the triangle: (1) AB = 9, BC = 4 and the third side, AC = 9 **OR** (1) AB = 9, BC = 4 and the third side AC = 4.

These two scenarios lead to two different perimeters for triangle ABC, HOWEVER, upon careful observation we see that the second scenario is an IMPOSSIBILITY. A triangle with three sides of 4, 4, and 9 is not a triangle. Recall that any two sides of a triangle must sum up to be greater than the third side. $4 + 4 < 9$ so these are not valid lengths for the side of a triangle.

Therefore the actual sides of the triangle must be AB = 9, BC = 4, and AC = 9. The perimeter is 22.

The correct answer is C.

5.
(1) SUFFICIENT: If we know that ABC is a right angle, then triangle ABC is a right triangle and we can find the length of BC using the Pythagorean theorem. In this case, we can recognize the common triple 5, 12, 13 - so BC must have a length of 12.

(2) INSUFFICIENT: If the area of triangle ABC is 30, the height from point C to line AB must be 12 (We know that the base is 5 and area of a triangle = 0.5 × base × height). There are only two possibilities for such such a triangle. Either angle CBA is a right triangle, and CB is 12, or angle BAC is an obtuse angle and the height from point C to length AB would lie outside of the triangle. In this latter possibility, the length of segment BC would be greater than 12.

The correct answer is A.

6.
By simplifying the equation given in the question stem, we can solve for x as follows:

$$\sqrt{x^8} = 81$$
$$x^4 = 81$$
$$x = 3$$

Thus, we know that one side of Triangle A has a length of 3.

Statement (1) tells us that Triangle A has sides whose lengths are consecutive integers. Given that one of the sides of Triangle A has a length of 3, this gives us the following possibilities: (1, 2, 3) OR (2, 3, 4) OR (3, 4, 5). However, the first possibility is NOT a real triangle, since it does not meet the following condition, which is true for all triangles: The sum of the lengths of any two sides of a triangle must always be greater than the length of the third side. Since 1 + 2 is not greater than 3, it is impossible for a triangle to have side lengths of 1, 2 and 3.

Thus, Statement (1) leaves us with two possibilities. Either Triangle A has side lengths 2, 3, 4 and a perimeter of 9 OR Triangle A has side lengths 3, 4, 5 and a perimeter of 12. Since there are two possible answers, Statement (1) is not sufficient to answer the question.

Statement (2) tells us that Triangle A is NOT a right triangle. On its own, this is clearly not sufficient to answer the question, since there are many non-right triangles that can be constructed with a side of length 3.

Taking both statements together, we can determine the perimeter of Triangle A. From Statement (1) we know that Triangle A must have side lengths of 2, 3, and 4 OR side lengths of 3, 4, and 5. Statement (2) tells us that Triangle A is not a right triangle; this eliminates the possibility that Triangle A has side lengths of 3, 4, and 5 since any triangle with these side lengths is a right triangle (this is one of the common Pythagorean triples). Thus, the only remaining possibility is that Triangle A has side lengths of 2, 3, and 4, which yields a perimeter of 9.

The correct answer is C: BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

7.
According to the Pythagorean Theorem, in a right triangle $a^2 + b^2 = c^2$.

(1) INSUFFICIENT: With only two sides of the triangle, it is impossible to determine whether $a^2 + b^2 = c^2$.

(2) INSUFFICIENT: With only two sides of the triangle, it is impossible to determine whether $a^2 + b^2 = c^2$.

(1) AND (2) SUFFICIENT: With all three side lengths, we can determine if $a^2 + b^2 = c^2$. It turns out that $17^2 + 144^2 = 145^2$, so this is a right triangle. However, even if it were not a right triangle, this formula would still be sufficient, so it is unnecessary to finish the calculation.

The correct answer is C

8.
The third side of a triangle must be *less* than the *sum* of the other two sides and *greater* than their difference (i.e. $|y - z| < x < y + z$).

In this question:

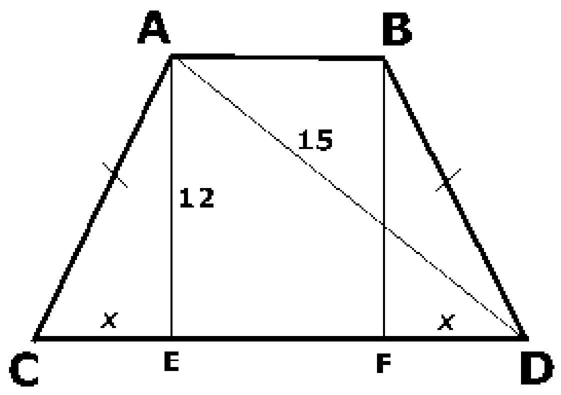
$|BC - AC| < AB < BC + AC$
$9 - 6 < AB < 9 + 6$
$3 < AB < 15$

Only 13.5 is in this range. $9\sqrt{3}$ is approximately equal to 9(1.7) or 15.3.

The correct answer is C.

9. The area of triangle ABD = (1/2)bh = (1/2)(6) h
The area of trapezoid BACE = (1/2)(6 + 18) h
Ratio = 6/24 = ¼

10.



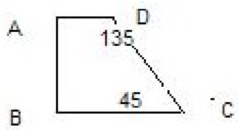
By sketching a drawing of trapezoid ABDC with the height and diagonal drawn in, we can use the Pythagorean theorem to see the ED = 9. We also know that ABDC is an isosceles trapezoid, meaning that AC = BD; from this we can deduce that CE = FD, a value we will call x. The area of a trapezoid is equal to the average of the two bases multiplied by the height. The bottom base, CD, is the same as CE + ED, or x + 9. The top base, AB, is the same as ED – FD, or 9 – x.

Thus the average of the two bases is $\frac{(x+9)+(9-x)}{2}=\frac{18}{2}=9$.

Multiplying this average by the height yields the area of the trapezoid: $9\times12=108$.

The correct answer is D.

11.



The figure can fulfill the entire requirement, but there is no any angle that equal to 60.

Sum of 4 angles = (n - 2) * 180 = 360
From 1: sum of the remaining angles are 360 - 2*90 = 180
From 2: either x + 2x = 180 => x = 60
Or x = 90/2 = 45 and y = 180 - 45 = 135.
Answer is E

12.
Given that line *CD* is parallel to the diameter, we know that angle *DCB* and angle *CBA* are equal.
Thus $x = 30°$.
First, let's calculate the length of arc *CAE*. Since arc *CAE* corresponds to an inscribed angle of 60° ($2x = 2\times30° = 60°$), it must correspond to a central angle of 120° which is 1/3 of the 360° of the circle. Thus we can take 1/3 of the circumference to give us the arc length *CAE*. The circumference is given as $2\pi r$, where r is the radius. Thus the circumference equals 10π and arc length *CAE* equals $(10/3)\pi$.

Now we need to calculate the length of *CB* and *BE*. Since they have the same angle measure, these lengths are equal so we can just find one length and double it. Let us find the length of *CB*. If we draw a line from *A* to *C* we have a right traingle because any inscribed triangle that includes the diameter is a right triangle. Also, we know that $x = 30°$ so we have a 30-60-90 triangle. The proportions of the length of the sides of a 30-60-90 triangle are 1- $\sqrt{3}$ -2 for the side opposite each respective angle. We know the hypotenuse is the diameter which is $2*r = 10$. So length *AC* must equal 5 and length *CB* must equal 5 $\sqrt{3}$.

Putting this all together gives us $(10/3)\pi + 2 \times 5\sqrt{3} = (10/3)\pi + 10\sqrt{3}$

The correct answer is D.

13.

They're asking whether the angle at Y is a right angle. Even if you have the two statements together, you only know that the angles at X (from statement 1) and at Z (from statement 2) are right angles. This isn't good enough; the angles at W and Y can be any two angles that add to 180 degrees. Should be (E).

14.

Ans. 6

'intersection' just means 'shared points'.

1 - Triangle is out side the circle and tangential

2 - Circle divides one side of triangle in 3 parts. Intersects only one side at 2 points.

3 - Inscribed circle. All sides of triangle tangent to circle.

4 - Circle intersects only 2 sides of triangle, 2 points each side.

5 - Circle intersects 2 sides , at 2 points each, and is tangential to the third side.

6 - Circle intersects all 3 sides, each of them at 2 points

intersection means any common points.

15.

(1) COD is 60, you all ready know that OB and OC are equal because they are both radii of the circle. I labeled angles CBO and BCO s. Since it is given that OC is equal to AB you know that AB is also then equal to OB. So I labeled BAO and AOB both x since they are equal due to the opposite sides being equal. By the rule of exterior angles of a triangle x + x = s, so 2x = s. I labeled angle BOC t. x+t = 120 (180 - COD (60) = 120). So for the larger triangle I have the equation x + s + x+t = 180. I substitute 120 for x+t and 2x for x which gives me x + 2x + 120 = 180. Subtract 120 from both sides and you get 3x = 60, so x = 20, SUFFICIENT.

(2) BCO is 40. Using the same descriptors for angles I have and utilizing again the exterior angles rule I have x + x = s so 2x = 40, x = 20, SUFFICIENT.

Answer D.

16.

First Statement: If T is 100 degrees, it cannot be one of the equal angles of the isoceles triangle...because 100 + 100 = 200 > 180, even before taking into account the third angle. So the remaining two angles have to be the equal ones i.e. 180 - 100 = 80 / 2 = 40 = R = S. SUFFICIENT

Second Statement: S=40. From this we cannot be sure if S is one of the equal angles. If S is NOT one of the equal angles, then S=40 and R=T=70 (180-40 = 140/2). If S was one of the equal angles then, S=40 = R or T i.e. If R = 40, then T = 110 OR if T = 40, then R = 110. INSUFFICIENT
first of all, **WE DON'T KNOW WHICH TWO ANGLES ARE EQUAL**. there are two possibilities for an isosceles triangle with a 40° angle in it:
(case 1) 40°, 40°, 100° (if angle S = 40° is one of the two equal angles)
(case 2) 40°, 70°, 70° (if angle S = 40° is NOT one of the two equal angles)

worse yet - **it would still be insufficient even if only case (1) were possible!**
this is because there are two DIFFERENT angles - 100° and the other 40° - remaining, and *you don't know which of these is angle T.* i.e., angle T could still be either 40° or 100° in this case.
Ans. A

17.
From Statement 1:
Angle QPR = 30 as shown.

Now, 30 + x + y = 90
and , x + z = 90

subtracting the 2 above, gives z-y=30 (Remember, we have to find the difference of the angles, not the actual values of each of the angles)

Statement 2 is same as Statement 1.

PRS = QPR + PQR (exterior angle of a triangle is equal to sum of two interior angles)
PRS - PQR = QPR = 30. (1) is sufficient.
(2) provides the same information as (1).

Hence, (D).

in this problem:
let angle QPR = 30°
let angle RPS = x°

then
using triangle PRS, we have x + 90 + angle PRS = 180°
so, angle PRS = (90 - x)°

using triangle PQS, we have (x + 30) + 90 + angle PQS = 180°

so, angle PQS = (60 - x)°

now (90 - x) is greater than (60 - x) by a margin of exactly thirty, so this is sufficient to answer the problem.

18.
statement (1)
since angle X is bigger than angle Y, it follows that segment PQ is steeper (i.e., has a greater slope) than segment RS.

imagine drawing perpendiculars (which in this diagram would be vertical lines) down from P and S, and considering the right triangles thereby formed.
the vertical legs of those right triangles would have the same length, because they're drawn between the same parallel lines.
the horizontal leg of the triangle with hypotenuse PQ would be shorter, though, because the slope (= rise/run) is greater. since "rise" is identical, as just mentioned, the fact that (rise/run) is greater means that "run" must be smaller.
because the vertical legs have the same length and the horizontal leg of the left-hand triangle is shorter, it follows that the left-hand hypotenuse (i.e., PQ) is shorter.
sufficient.

(2)
this statement is symmetric in x and y, meaning that you can switch x and y without consequence.
consider two cases in which this happens: say, x = 40 and y = 60, and then x = 60 and y = 40.
in the latter case, the reasoning is the same as for statement (1); in the former case, it's the opposite, and PQ is now longer.
insufficient.

answer = A
19.
First off, note that the conditions given in statements (1) and (2), individually, are identical. (i.e., if you flip the triangle around, statement 1 becomes statement 2, and vice versa.). That serves to eliminate choices a and b in a hurry: if statement (1) is sufficient then statement (2) must be as well, and vice versa. That leaves us with the last 3 choices. You can visualize the fact that one of the two statements alone won't do the job: Imagine that statement (1) alone is true, making triangle QRS isosceles. That means segment QS is fixed in place. However, there are no restrictions on triangle STU. That means, in effect, that we can move point U wherever we feel like moving it. As we 'slide' point U along the bottom of the triangle, the value of x changes; therefore, statement (1) alone (and hence statement (2) alone) is insufficient. If you don't buy the above argument, or if it's just something you'd never possibly think of within the time limit, then you could always try plugging in numbers and seeing that x can have different values.
Statements (1) and (2) together:
since the triangle is a right triangle, we know that angles R and T must add to 90 degrees. Let angle R be y degrees, and let angle T be (90 - y) degrees.

Then
each of angles RQS and RSQ is (180 - y)/2 = 90 - y/2 degrees; and
each of angles TSU and TUS is (180 - (90 - y))/2 = 45 + y/2 degrees.
therefore, since angle RSQ, x, and angle TSU make a straight line together,
x = 180 - RSQ - TSU
= 180 - (90 - y/2) - (45 + y/2)
= 45 degrees.
sufficient answer = C

20.
There are only two arc-angle relationships that the gmat expects you to know:

Inscribed angle (with the vertex on the circle itself): the arc that gets cut off by the angle has **twice** as many degrees as does the angle.

Central angle (with the vertex at the center of the circle): the arc that gets cut off by the angle has the **same** number of degrees as does the angle.

In this case, angle QPR and angle PRO are both inscribed angles. They are also alternate interior angles (the 'Z angles' formed by parallel lines and a transversal), so both are 35 degrees. Therefore, arc OP and arc QR are both 70 degrees each. Since OPQR is a semicircle, it contains a total of 180 degrees, so arc PQ is 180 - 70 - 70 = 40 degrees.

40 degrees is 1/9 of a circle, so that arc is 1/9 of the total circumference of the circle, or (1/9)(18pi) = 2pi.

Answer = A